1 (i) AE = $\sqrt{(15^2 + 20^2 + 0^2)} = 25$	M1 A1 [2]	
(ii) $\overrightarrow{AE} = \begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix} = 5\begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$ Equation of BD is $\mathbf{r} = \begin{pmatrix} -1 \\ -7 \\ 11 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$ BD = 15 $\Rightarrow \lambda = 3$ $\Rightarrow$ D is (8, –19, 11)	M1 A1 M1 A1cao [4]	Any correct form or $\mathbf{r} = \begin{pmatrix} -1 \\ -7 \\ 11 \end{pmatrix} + \lambda\begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix}$ λ = 3 or 3/5 as appropriate
(iii) At A: $-3 \times 0 + 4 \times 0 + 5 \times 6 = 30$ At B: $-3 \times (-1) + 4 \times (-7) + 5 \times 11 = 30$ At C: $-3 \times (-8) + 4 \times (-6) + 5 \times 6 = 30$ Normal is $\begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$	M1 A2,1,0 B1 [4]	One verification (OR B1 Normal, M1 scalar product with 1 vector in the plane, A1two correct, A1 verification with a point OR M1 vector form of equation of plane eg r=0i+0j+6k+μ(i+7j-5k)+ν(8i+6j+0k) M1 elimination of both parameters A1 equation of plane B1 Normal *)
(iv) $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} . \overrightarrow{AE} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} . \begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix} = 60 - 60 = 0$ $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} . \overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \begin{pmatrix} -1 \\ -7 \\ 5 \end{pmatrix} = -4 - 21 + 25 = 0$ $\Rightarrow \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ is normal to plane Equation is $4x + 3y + 5z = 30$.	M1 E1 M1 A1 [4]	scalar product with one vector in plane = 0 scalar product with another vector in plane = 0 $4x + 3y + 5z = \ldots$ 30 OR as * above OR M1 for subst 1 point in 4x+3y+5z= ,A1 for subst 2 further points =30 A1 correct equation , B1 Normal
(v) Angle between planes is angle between normals $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$ $\cos\theta = \dfrac{4 \times (-3) + 3 \times 4 + 5 \times 5}{\sqrt{50} \times \sqrt{50}} = \dfrac{1}{2}$ $\Rightarrow \theta = 60°$	M1 M1 A1 A1 [4]	 Correct method for any 2 vectors their normals only (rearranged) or 120° cao

Question			Answer	Marks	Guidance
2	**(i)**		AC = cosec θ $\Rightarrow$ AD = cosecθ sec φ	M1 A1 **[2]**	or 1/sin θ oe but not if a fraction within a fraction
2	**(ii)**		DE = AD sin ($\theta + \varphi$) = cosec θ sec φ sin ($\theta + \varphi$) = cosecθ sec φ(sin θ cos φ + cos θ sin φ) $= \frac{\sin\theta\cos\varphi + \cos\theta\sin\varphi}{\sin\theta\cos\varphi}$ $= 1 + \frac{\cos\theta}{\sin\theta}\frac{\sin\varphi}{\cos\varphi}$ = 1 + tan φ / tan θ * …………………………………………………….. **OR** equivalent, eg from DE = CB + CD cos θ = 1 + CD cos θ = 1 + AD sin φ cos θ = 1 + cosec θ sec φ sin φ cos θ = 1 + tanφ /tanθ*	M1 M1 A1 M1 M1 A1 **[3]**	AD sin($\theta + \varphi$) with substitution for their AD correct compound angle formula used **Do not** award both M marks unless they are part of the same method. (They may appear in either order.) simplifying using tan = sin/cos. A0 if no intermediate step as **AG** …………………………………………………………………………………….. from triangle formed by using X on DE where CX is parallel to BE to get DX = CD cos θ and CB = 1 (oe trigonometry) substituting for both CD = AD sinφ and their AD oe to reach an expression for DE **in terms of θ and φ only** (M marks must be part of same method) **AG** simplifying to required form

3			$\overrightarrow{AB}=\begin{pmatrix}-1\\2\\1\end{pmatrix}, \overrightarrow{AC}=\begin{pmatrix}-2\\-4\\0\end{pmatrix}$	M1	scalar product with any two directions in the plane (BC=$\begin{pmatrix}-1\\-6\\-1\end{pmatrix}$)
			$\mathbf{n}.\overrightarrow{AB}=\begin{pmatrix}2\\-1\\4\end{pmatrix}.\begin{pmatrix}-1\\2\\1\end{pmatrix}=2\times(-1)+(-1)\times 2+4\times 1=0$	B1	evaluatio needed
			$\mathbf{n}.\overrightarrow{AC}=\begin{pmatrix}2\\-1\\4\end{pmatrix}.\begin{pmatrix}-2\\-4\\0\end{pmatrix}=2\times(-2)+(-1)\times(-4)+4\times 0=0$	B1	evaluatio needed
			$\Rightarrow$ **n** is perpendicular to plane.		thus finding the scalar product with only one direction vector is M0 B1 B0. No marks for scalar product with position vectors. or SC finding direction of normal vector by using vector cross product, M1A1 eg 4*i*-2*j*+8*k* and showing this is a multiple of 2*i*-*j*+4*k*, A1
			Equation of plane is **r**.**n** = **a**.**n** $\Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}2\\-1\\4\end{pmatrix}=\begin{pmatrix}2\\0\\1\end{pmatrix}.\begin{pmatrix}2\\-1\\4\end{pmatrix}$	M1	For any complete method leading to the cartesian equation of the plane eg from vector form and eliminating parameters (there are many possibilities eg $r=\begin{pmatrix}2\\0\\1\end{pmatrix}+\mu\begin{pmatrix}-1\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}-2\\-4\\0\end{pmatrix}$ $x=2-\mu-2\lambda, y=2\mu-4\lambda,$ $z=1+\mu$, $2x-y=4-4\mu=4-4(z-1)=8-4z$, $2x-y+4z=8$ gets M1 once the parameters have been eliminated.
			$\Rightarrow \quad 2x - y + 4z = 8$	A1	o
				[5]	**SC1** If they say the plane is of the form $2x-y+4z=c$ and then show all points satisfy $2x-y+4z=8$ they can have M1 A1 for the first point and B2 for both the others. **SC2** If they omit verification and find equation from vector form without using normal as above and then state *2i*-*j*+4*k* is perpendicular they can get M1A1B2

4			$\begin{pmatrix} 4+3\lambda \\ 2 \\ 4+\lambda \end{pmatrix} = \begin{pmatrix} -1-\mu \\ 4+\mu \\ 9+3\mu \end{pmatrix}$		
			$\Rightarrow$ $4 + 3\lambda = -1 - \mu$ (1)	M1	equatin components
			$2 = 4 + \mu$ (2)		
			$4 + \lambda = 9 + 3\mu$ (3)		
			(2) $\Rightarrow \mu = -2$	B1	$\mu = -2$
			(1) $\Rightarrow 4 + 3\lambda = -1 + 2 \Rightarrow \lambda = -1$	A1	$\lambda = -$
			(3) $\Rightarrow 4 + (-1) = 9 + 3 \times (-2)$ so consistent	A1	checking third component
			Point of intersection is (1, 2, 3)	A1	dependent on all previous marks being obtained
				[5]	

5(i) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$ $\overrightarrow{AB}.\overrightarrow{BC} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}.\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = 2\times5+3\times0+(-5)\times2=0$ $\Rightarrow$ AB is perpendicular to BC.	B1 B1 M1E1 [4]	
(ii) AB = $\sqrt{(2^2 + 3^2 + (-5)^2)} = \sqrt{38}$ BC = $\sqrt{(5^2 + 0^2 + 2^2)} = \sqrt{29}$ Area = ½ × $\sqrt{38}$ × $\sqrt{29}$ = ½ $\sqrt{1102}$ or 16.6 units^2	M1 B1 A1 [3]	complete method ft lengths of both AB, BC oe www

6	**(i)**	$x = -5 + 3\lambda = 1 \Rightarrow \lambda = 2$ $y = 3 + 2 \times 0 = 3$ $z = 4 - 2 = 2$, so (1, 3, 2) lies on 1st line. $x = -1 + 2\mu = 1 \Rightarrow \mu = 1$ $y = 4 - 1 = 3$ $z = 2 + 0 = 2$, so (1, 3, 2) lies on 2nd line.	M1 E1 E1 **[3]**	finding λ or μ verifying two other coordinates for line 1 verifying two other coordinates for line 2
	(ii)	Angle between $\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$ $\cos\theta = \dfrac{3\times2+0\times(-1)+(-1)\times0}{\sqrt{10}\sqrt{5}}$ $= 0.8485\ldots$ $\Rightarrow$ $\theta = 31.9°$	M1 M1 A1 A1 **[4]**	direction vectors only allow M1 for any vectors or 0.558 radians